VAPOR FLOW IN LOW-TEMPERATURE HEAT PIPES

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The authors consider the influence of nonuniformity and asymmetry of the heat supply and removal in the evaporation and condensation zones of heat pipes on the characteristics of the vapor flow.

In optimizing the scale and weight characteristics of heat pipes one meets the problems of limiting decrease of the cross section of the vapor channel, which leads unavoidably to a sharp increase of the pressure loss in the vapor flow. The influence of the vapor flow characteristics on the operating and limiting characteristics of low-temperature heat pipes here can be appreciable, and sometimes a govering factor. Thus, we need reliable theoretical relations to determine the vapor flow characteristics in heat pipes. This matter is complicated appreciably by the fact that in most cases in practice the heat flux is nonuniform along the evaporation and condensation zones, and for flat-plate heat pipes it is also unsymmetrical with respect to the longitudinal axis. The influence of nonuniformity of the heat flux in the evaporation and condensation zones of cylindrical heat pipes on their operation and limiting characteristics was investigated in detail in [1].

In this paper we consider the influence of nonuniformity and asymmetry of the heat supply and removal in the evaporation and condensation zones on the vapor flow characteristics in flat-plate heat pipes.

We consider the case of heat supply or removal according to a power law. On the one side we have

$$q^{\mathrm{I}} = q_1\bar{x}^n, \tag{1}$$

and on the other

$$q^{\mathrm{II}} = q_2\bar{x}^n, \tag{2}$$

where $\bar{x}$ is the dimensionless axial coordinate. For the sake of being definite we assume that $0 \leq q_2 \leq q_1$.

The case $q_2 = 0$ corresponds to a one-sided heat supply or heat removal, and the case $q_2 = q_1$ is symmetric. For $n = 0$ the heat flux is uniform along the zones considered. Figure 1 shows the coordinate system for the problem examined.

We write the system of equations describing the flow of incompressible vapor with constant physical properties

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right); \tag{3}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right); \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

with the boundary conditions

$$y = 0,\ v = \pm v_1\bar{x}^n;\ u = 0; \tag{6}$$

$$y = \delta,\ v = \mp v_2\bar{x}^n;\ u = 0; \tag{7}$$

$$x = 0,\ x = L;\ u = v = 0. \tag{8}$$

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Here and below the upper sign of the chosen coordinate system refers to the evaporation zone, and the lower sign refers to the condensation zone.

We arrive at the dimensionless variables:

$$\bar{x} = x/L_e \text{ for the evaporator} \tag{9}$$

$$\bar{x} = 1 - x/L_c \text{ for the condenser} \tag{10}$$

$$\eta = y/\delta.$$

We introduce a certain velocity function f in the form

$$f = \frac{v}{\pm v_1 \bar{x}^n} = \frac{\delta v}{\nu \, \mathrm{Re}_n \bar{x}^n}, \tag{11}$$

where

$$\mathrm{Re}_n = \frac{\pm v_1 \delta}{\nu}. \tag{12}$$

Here for the evaporator $\mathrm{Re}_n > 0$, and for the condenser $\mathrm{Re}_n < 0$ the velocity v_1 is determined from the expression

$$v_1 = \frac{q_1}{\rho r}. \tag{13}$$

To solve the system of equations (3)-(5) we use the method of perturbation theory [2]. As the perturbing parameter here it is convenient to use the Reynolds number, expressed in terms of the normal velocity of the vapor, Eq. (12).

We consider the case $|\mathrm{Re}_n| \ll 1$. We expand the function f in a series in the perturbing parameter:

$$f = f_0 + f_1 \mathrm{Re}_n \bar{x}^1 + \ldots + f_m \mathrm{Re}_n^m \bar{x}^m. \tag{14}$$

It is assumed that the function f_i, where i = 0, 1, 2, ..., is independent of the axial coordinate $\bar{x}$. To calculate the dependence of the function f on the axial coordinate we also introduce $\bar{x}$ in the series.

We substitute Eqs. (9), (10), (11), and (14) into Eq. (3) and (4), take account of Eq. (5), differentiate Eq. (3) with respect to y and Eq. (4) with respect to x, equate the right sides, evaluate the terms of the differential equation obtained analogously as in [1], gather terms with the same power of Re_n, equate them to zero, and obtain the following system of ordinary differential equations of fourth order for the functions f_i:

$$f_0^{\mathrm{IV}} = 0; \tag{15}$$

$$f_1^{\mathrm{IV}} = \frac{2n+1}{n+1}(f_0 f_0''' - f_0' f_0''); \tag{16}$$

$$f_2^{\mathrm{IV}} = \frac{3n+1}{2n+1}(f_0 f_1''' - f_0'' f_1') + \frac{3n+1}{n+1}(f_1 f_0''' - f_1'' f_0'); \tag{17}$$

$$f_m^{\mathrm{IV}} = \sum_{i=0}^{m-1} A_i (f_i f_{m-1-i}''' - f_i'' f_{m-1-i}'), \tag{18}$$

where

$$A_i = \frac{(m+1)n+1}{(m-i)n+1}; \; m = 1, 2, 3, \ldots$$

with the boundary conditions

$$\eta = 0 \quad f_0 = 1, \; f_{i+1} = 0, \; f_i' = 0; \tag{19}$$

$$\eta = 1 \quad f_0 = \mp v_2/v_1, \; f_{i+1} = 0, \; f_i' = 0, \; i = 0, 1, 2, 3, \ldots \tag{20}$$

The equation obtained, Eq. (15), for the unperturbed flow (zero order approximation) corresponds to flow in a smooth-walled pipe. The functions f_1 (first approximation), f_2

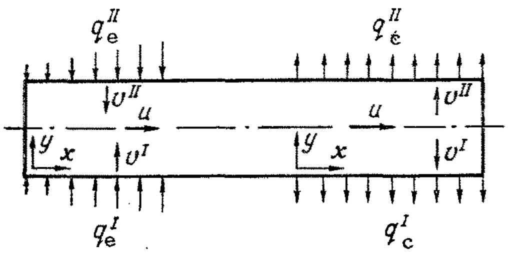

Fig. 1

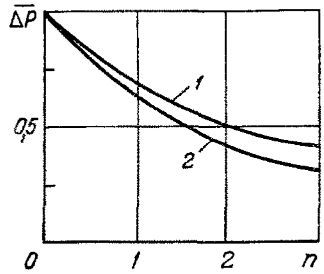

Fig. 2

Fig. 1. The coordinate system.

Fig. 2. Influence of the heat-flux distribution on the pressure loss $\overline{\Delta P} = \Delta P/\Delta P_{n=0}$ in the vapor channel for $Re_n = 0.2$ and $\bar{x} = 1$: 1) evaporator; 2) condenser.

(second approximation), etc. account for the influence of the normal velocity component of the vapor at the phase transition surface. The coefficients A_i account for the influence nonuniformity along the normal velocity component of the vapor at the phase transition surface. Integration of Eqs. (15), (16), (17), and (18) and implementation of the boundary conditions (19) and (20) do not present any particular complexity.

According to an estimate of accuracy in [2], it was shown in [1] that to sufficient accuracy one can restrict the solution to the first approximation for the function f. Even with $Re_n = 1$ the error in determining the function f does not exceed one percent.

Thus, we have:

$$f = 1 - 3N\eta^2 + 2N\eta^3 + \frac{2n+1}{n+1} N \, \mathrm{Re}_n \bar{x}^n \left[\left(\frac{1}{2} - \frac{8}{35} N \right) \eta^2 - \left(1 - \frac{27}{70} N \right) \eta^3 + \frac{1}{2}\eta^4 - \frac{3}{10} N\eta^5 + \frac{1}{5} N\eta^6 - \frac{2}{35} N\eta^7 \right], \tag{21}$$

where $N = 1 \pm v_2/v_1$.

Using Eq. (21) we obtain an expression to determine the pressure drop in the evaporation and condensation zones

$$\Delta P = -\rho \left(\frac{\nu L}{\delta^2} \right)^2 \frac{\mathrm{Re}_n \bar{x}^{n+2}}{(n+1)(n+2)} N \left[12 + \frac{81}{35} N \mathrm{Re}_n \frac{n+2}{2n+2} \bar{x}^n - \frac{n+2}{3n+2} \mathrm{Re}_n^2 \bar{x}^{2n} \left(1 - \frac{16}{35} N \right) \right], \tag{22}$$

where $Re_n > 0$ for the evaporation zone and $Re_n < 0$ for the condensation zone.

By analyzing Eq. (22) we see that for one-sided heat supply or removal ($q_2 = 0$, $N = 1$) with q_1 = const the pressure losses in the evaporation and condensation zones decrease appreciably. This can be explained by the decrease of mass flow through the zones. But if, while decreasing q_2 we proportionally increase q_1, i.e., we keep $q_1 + q_2$ = const then the pressure losses in the vapor flow of the evaporation and condensation zones are practically unchanged.

As can be seen from Eq. (22) the pressure losses in the vapor flow (this holds without complication also for a liquid, using existing relations to determine the pressure gradients) are appreciably affected by the heat load distribution along the evaporation and condensation zones.

When the heat flux is concentrated into the transport zone the pressure losses decrease sharply (Fig. 2). The reason is that the main mass of heat transfer agent traverses less distance along the zones. When the heat flux is concentrated at the ends of the heat pipe one would expect the reverse to be true. The shape of the surface of the evaporation and condensation zones appreciably influences the shear stress at the phase interphase. For example, for flat-plate pipes in the evaporation zones with increase of heat supply the shear stresses become less than for flow in channels with impermeable walls, but become

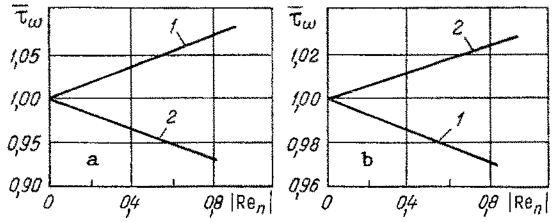

Fig. 3. Influence of evaporation and condensation on the shear stresses at the vapor–liquid interface: 1) evaporator; 2) condenser; a) cylindrical, b) flat-plate channel.

greater in the condensation zone (Fig. 3). In cylindrical heat pipes, as shown by the results of the calculations of [1], the opposite picture is observed.

A comparison with the experimental results of [3, 4] shows that the solutions obtained agree well with experiment in the range $|Re_n| < 1.2$. For $Re_n = 3.72$ the divergence of the results calculated using Eq. (22) from the experimental data of [4], which investigated a flat-plate heat pipe with one-sided heat supply and removal, reaches 27%.

NOTATION

ρ, density, kg/m^3; ν, kinetmatic viscosity, m^2/sec; δ, distance between the phase interface surfaces, m; L, length, m; v, u, normal and axial comonents of the vapor velocity, m/sec; P, pressure, N/m^2; q, specific heat flux, W/m^2; $\bar{\tau}_w = \tau/\tau_{i.w}$. Subscripts: e, evaporation; c, condensation; n, normal to the surface; i.w, impermeable wall.

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